5. Races

Time, Speed & Distance — Study Notes (Part 5 of 6)

Race Vocabulary — Linear Tracks

- A beats B by 10 m: when A finishes, B is still 10 m behind.
- A beats B by 10 s: A finishes in t seconds; B needs t + 10 seconds.
- A gives B a head start of 10 m in a 100 m race: A runs the full 100 m; B only needs to run 90 m (B starts 10 m ahead).
- A gives B a head start of 10 s: B starts running 10 seconds before A does.
- Dead heat: both A and B finish at exactly the same time.

Circular Tracks

A single lap on a circular track behaves exactly like a linear race. The interesting problems come from repeated laps.

Same direction: Time to meet = Track Length / (S1 − S2)
Opposite direction: Time to meet = Track Length / (S1 + S2)

Worked Example — Opposite Directions

Track length 600 m, speeds 120 m/s and 180 m/s, opposite directions
Relative speed = 120 + 180 = 300 m/sec → they meet every 600/300 = 2 seconds.
Number of distinct meeting points = ratio of speeds added: 120:180 = 2:3 → 2+3 = 5 points

Worked Example — Same Direction

Track length 100 m, speeds 40 m/s and 10 m/s, same direction
Relative speed = 40 − 10 = 30 m/sec → they meet every 100/30 = 10/3 seconds.
Number of distinct meeting points = ratio of speeds subtracted: 40:10 = 4:1 → 4−1 = 3 points

💡 *Opposite direction → ADD the ratio terms for the number of meeting points. Same direction → SUBTRACT the ratio terms.*

Worked Examples — Linear Races

Example — A beats B by time (using race-splitting)
A finishes a 100 m race in 36 s, B in 45 s. By how much distance does A win?
Ratio of times 36:45 = 4:5, so ratio of speeds = 5:4. In the time A finishes (100 m), B covers (100/5)×4 = 80 m.
A wins by 100 − 80 = 20 m

Example — Chained Comparison (Zigzag Method)
A beats B by 20 m in a 100 m race. In a separate 100 m race, B beats C by 4 m. By how much does A beat C over 100 m?
When B covers 100 m, C covers 96 m → so for every 25 m B covers, C covers 1 m less; scaling: 75 m → 3 m less → C covers 72 m when B covers 75 m.
So when A covers 100 m (B is at 75 m), C is at 72 m. A beats C by 100 − 72 = 28 m

Example — Circular Track (using circumference)

Two boats move on a circular lake of diameter 4 km at 3 km/hr and 1 km/hr, opposite directions. Find time to meet.
Circumference = $2\pi r = 2 \times (22/7) \times 2 = 4\pi$ km. Relative speed = 3+1 = 4 km/hr.
Time = $4\pi / 4 = \pi \approx 3.14$ hrs

✎ Practice Questions

1. On a circular track of length 4500 m, two runners move in the same direction at 54 km/hr and 36 km/hr. Find the time until they first meet.
2. A beats B by 20 m in a 100 m race, finishing 5 seconds ahead. Find B's speed and the time A takes to finish.
3. In a 750 m race, Arun gives Darani a head start of 250 m. Their speeds are in the ratio 1:3 (Arun:Darani). Who wins, and by how much?
4. A gives B a head start of 7 m. If A can give B a start of 1 m for every 5 m of race length (to finish in a dead heat), find the total length of the race.
5. Ani and Bini run a 300 m race. When Ani finishes, Bini is 22.5 m behind, and Bini takes 6 seconds to cover that gap. Find Bini's time to finish the full 300 m race.

✓ Answer Key

1. 900 seconds
2. B's speed = 4 m/s; A finishes in 20 seconds
3. Darani wins by 250 m
4. 35 m
5. 80 seconds