QUANTITATIVE APTITUDE
NUMBER SYSTEM

COMPLETE STUDY MATERIAL

Consolidated Concepts, Worked Examples & Practice Sums

Unit 1	Classification of Numbers
Unit 2	Divisibility Tests
Unit 3	Power Cycles & Remainder Cycles
Unit 4	Factors, Multiples & Perfect Numbers
Unit 5	Applications of HCF & LCM
Unit 6	Solved Problem Bank — Mixed Applications

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1 Classification of Numbers

Numbers used in mathematics can be organised into a family tree. At the very top sit Complex Numbers, written as a + ib, where 'a' is the Real part and 'ib' is the Imaginary part.

COMPLEX NUMBER RULE

$$a + ib \quad (a = \text{Real part, } b = \text{coefficient of the Imaginary part})$$
$$\text{If } a = 0 \rightarrow \text{ the number is purely Imaginary (ib)}$$
$$\text{If } b = 0 \rightarrow \text{ the number is purely Real (a)}$$

1.1 Imaginary Numbers

Imaginary numbers do not correspond to any real-world quantity — you can never physically count '5i' apples. They are built from i, defined so that $i^2 = -1$.

POWERS OF I

$$i^1 = i$$
$$i^2 = -1$$
$$i^3 = i^2 \times i = -i$$
$$i^4 = i^2 \times i^2 = 1 \quad \text{(then the cycle repeats every 4 powers)}$$

NOTE *Because the cycle length is 4, the sum of any 4 consecutive powers of i is always 0 ($i + i^2 + i^3 + i^4 = 0$).*

EXAMPLE

Simplify $(i^{17}+i^{18}+i^{19}+i^{20}) / (i^{52}+i^{53}+i^{54}+i^{55})$

Any 4 consecutive powers of i sum to 0, so both numerator and denominator are 0.

Result = 0/0 → undefined.

1.2 Real Numbers

Real numbers are numbers that exist in nature and can be placed exactly on a number line. They split further into Rational and Irrational numbers.

Rational Numbers

A number is rational if it can be written as a definite, exact fraction p/q — equivalently, if every digit at every position of the decimal expansion is predictable, even if the decimal never ends.

EXAMPLE

8 = 8/1 → rational

0.75 = 3/4 → rational

8.33 = 833/100 → rational (Non-Recurring, Terminating — 'NRT')

Converting a Recurring Decimal to a Fraction

METHOD

1. Let x = the recurring decimal.
2. Multiply x by a power of 10 that shifts the decimal point past one full repeating block.
3. Subtract the original equation from the new one to cancel the repeating part.
4. Solve for x as a fraction.

EXAMPLE

Show that 87.435435435… is rational.

x = 87.435435… → (1)

1000x = 87435.435435… → (2)

(2) − (1): 999x = 87348 → x = 87348/999 (a definite p/q) → Rational (this type is 'NTR': Non-Terminating, Recurring)

Irrational Numbers

A number is irrational if it cannot be written as a definite p/q — its decimal digits never terminate and never settle into a repeating pattern, so you can never predict a specific digit position.

EXAMPLE

√2 = 1.414213… → the digits never repeat → Irrational

π = 3.141592653589793… → Irrational (22/7 is only an approximation of π, and 22/7 itself IS rational since it terminates as a fraction)

Integers & Prime / Composite Numbers

- Integers — whole numbers with no fractional part, positive or negative (…, −2, −1, 0, 1, 2, …)
- Prime Number — a positive number with exactly two divisors: 1 and itself.
- Composite Number — a positive number with more than two divisors (built from smaller factors, e.g. 12 = 2 × 3 × 2).
- 1 is neither prime nor composite — it has only ONE divisor (itself), so it fails the 'exactly two divisors' test for prime, and there's nothing smaller to build it from, so it fails the composite test too.
- 0 is neither prime nor composite — every non-zero number divides 0, so it has infinitely many divisors.

Co-Prime Numbers

Two numbers are co-prime (or relatively prime) if their HCF is 1 — the only positive integer that divides both is 1. Co-prime numbers do NOT need to be prime themselves.

EXAMPLE

(2, 3) — both prime, HCF = 1 → co-prime

(3, 4) — 4 is not prime, but HCF(3,4) = 1 → still co-prime

Any two consecutive integers are always co-prime.

1.3 The Number Family Tree

Level	Category	Contains
1	Complex Numbers	a + ib — all numbers
2	Real Numbers (b = 0)	Numbers on the number line
3	Rational Numbers	p/q form — NRT or NTR decimals
3	Irrational Numbers	Non-terminating, non-recurring decimals (√2, π)
4	Integers	…,−2,−1, 0, 1, 2,… (subset of Rational)
4	Fractions	Non-integer rational numbers (subset of Rational)
5	Whole Numbers	0, 1, 2, 3, … (subset of Integers)
6	Natural Numbers	1, 2, 3, … (subset of Whole Numbers)

NOTE *Nesting rule: Natural ⊂ Whole ⊂ Integer ⊂ Rational ⊂ Real ⊂ Complex. So every whole number is automatically rational, and every integer is automatically real.*

1.4 Worked Examples

EXAMPLE

Classify: (i) 6543 (ii) √9 (iii) 0.13563487623… (iv) 0.57324354354354…

(i) 6543 → Rational (it's a whole number, definite value)

(ii) √9 = 3 → Rational (simplifies to a whole number)

(iii) Non-terminating, non-repeating pattern → Irrational (NTNR)

(iv) Non-terminating but the block '354' repeats → Rational (NTR)

EXAMPLE

Give an example of a number that is Real, Rational, Whole, Integer AND Natural.

Any natural number works, e.g. 1 — it satisfies every nested category simultaneously.

EXAMPLE

Give an example of a number that is Real and Irrational.

√3 = 1.732050… — never terminates, never repeats → Real & Irrational.

EXAMPLE

Give an example of a number that is Real, Rational and Non-Integer.

1/3 = 0.333… — a definite recurring fraction, but not a whole integer.

1.5 Practice Sums

1. Classify as rational or irrational, with reason: (a) 245 (b) −67 (c) −0.263 (d) 4783 (e) 0.32424… (f) 0 (g) π (h) √5

2. Convert the recurring decimal 4.872872872… into a fraction.

3. Give one example each of a number that is: (a) Real & Rational but not an Integer, (b) Real, Irrational (c) Integer but not Natural.

4. Are all co-prime pairs necessarily prime numbers? Justify with an example.

5. Simplify: $(i^{23} + i^{24} + i^{25} + i^{26}) \div (i^{100} + i^{101} + i^{102} + i^{103})$

2 Divisibility Tests

A number is divisible by another number if dividing them leaves no remainder — the result is a whole number. Divisibility rules let you check this instantly, without doing long division.

2.1 Why the Rules Work — The Core Idea

Every rule comes from expanding a number by place value (units, tens, hundreds…) and noticing that most place values are already divisible by the test number, leaving only a small part to actually check.

EXAMPLE

Why check only the last digit for divisibility by 2?

Any number = …+ (digit×100) + (digit×10) + (units digit)

100 and 10 are already divisible by 2, so only the units digit decides the outcome.

2.2 Divisibility Rules Reference Table

Divisor	Rule	Why (Key Insight)
2	Last digit is 0, 2, 4, 6 or 8	10 is already divisible by 2
3	Sum of digits divisible by 3	100, 10 leave remainder 1 each when ÷3
4	Last 2 digits divisible by 4	100 is already divisible by 4
5	Last digit is 0 or 5	10 is already divisible by 5
6	Divisible by both 2 AND 3 (co-primes)	6 = 2 × 3
8	Last 3 digits divisible by 8	1000 is already divisible by 8
9	Sum of digits divisible by 9	100, 10 leave remainder 1 each when ÷9
10	Last digit is 0	10, 100, 1000… are all divisible by 10
11	Difference of alternate digit-sums is 0 or a multiple of 11	100 leaves +1, 10 leaves −1 (alternating pattern)
12	Divisible by both 4 AND 3 (co-primes)	12 = 4 × 3

Divisor	Rule	Why (Key Insight)
18	Divisible by both 2 AND 9 (co-primes)	$18 = 2 \times 9$

NOTE *Golden Rule for composite divisors: split the divisor into two co-prime factors (factors that share no common divisor except 1) and test both. E.g. for 88, test 8 AND 11 (since 88 = 8 × 11 and they're co-prime) — NOT 8 and 22.*

2.3 Special Rule — Divisibility by 7

METHOD

1. Double the last digit.
2. Subtract it from the remaining leading number.
3. If the result is divisible by 7, so is the original number.

EXAMPLE

Check 343: last digit 3 → 3×2 = 6
34 − 6 = 28 → divisible by 7 → 343 IS divisible by 7.

2.4 Special Rule — Divisibility by 13

METHOD

1. Multiply the last digit by 4.
2. Add it to the remaining leading number.
3. If the result is divisible by 13, so is the original number.

EXAMPLE

Check 195: last digit 5 → 5×4 = 20
19 + 20 = 39 → divisible by 13 → 195 IS divisible by 13.

2.5 Special Case — Reversed Numbers and 9

If a number N is divisible by 9, its digit-reversal is also divisible by 9 (the same applies for 99, 999, …).

EXAMPLE

N = 459 → divisible by 9 (4+5+9=18)
Reversed: 954 → also divisible by 9 (9+5+4=18)

2.6 Worked Examples

EXAMPLE

Which numbers below are divisible by which? → 15, 27, 36, 268, 4518, 3619, 15000
15 → 3, 5 | 27 → 3, 9 | 36 → 2,3,4,6,9 | 268 → 2, 4
4518 → 2,3,6,9 | 3619 → 11 | 15000 → 2,3,4,5,6,8

EXAMPLE

A company made 87,195 packs of gum, split equally among flavours. Which of these could be the number of flavours: 9, 10, 5, 2?
Test each option against 87195 using divisibility rules — only 5 divides it exactly (last digit is 5).
Answer: 5 flavours.

EXAMPLE

Find the missing digit: 12473_ is divisible by 9
Known digit sum = 1+2+4+7+3 = 17. Need 17+_ divisible by 9 → next multiple of 9 after 17 is 18 → missing digit = 1.

EXAMPLE

Find the missing digit: 1245_42 is divisible by 11
Sum of digits at odd positions − sum at even positions = 0 or a multiple of 11.
Positions (right to left): 2,4,_,5,4,2,1 → alternate sums (4+_+4+1) − (2+5+2) = (9+_) − 9 = _
For this to be 0 or a multiple of 11, missing digit = 0 (or 11, but a single digit must be 0).

2.7 Practice Sums

1. Which is the only number that exactly divides 87,195 among the options 2, 3, 5, 9?
2. Find the missing digit K such that 36412K is divisible by 33.
3. Find the missing digit such that 12875_ is divisible by 8.
4. A number A8563145B is divisible by 88. Find A and B.
5. Find the number of integers between 1 and 1000 that are divisible by all of 6, 7 and 15.
6. 1234567875 is divided by 9. What is the remainder?

3 Power Cycles & Remainder Cycles

When a number is raised to increasing powers, its unit digit (or its remainder with a fixed divisor) doesn't change randomly — it repeats in a fixed cycle. Knowing the cycle length lets you jump straight to the answer for very large powers.

3.1 Unit Digit Power Cycles

Base digit	Behaviour	Cycle
0, 1, 5, 6	Always the same digit, for any power	constant (0/1/5/6)
2, 3, 7, 8	Repeats every 4 powers	2→2,4,8,6 | 3→3,9,7,1 | 7→7,9,3,1 | 8→8,4,2,6
4, 9	Repeats every 2 powers	4→4,6 (odd power=4, even power=6) | 9→9,1 (odd=9, even=1)

METHOD

1. Take the base's unit digit.
2. Find its cycle length (4, 2, or fixed).
3. Divide the exponent by the cycle length — find the remainder.
4. That remainder tells you the position in the cycle (remainder 0 means the LAST position in the cycle).

EXAMPLE

Last digit of 7^{34}? Cycle of 7 = 7,9,3,1 (length 4)
34 ÷ 4 → remainder 2 → 2nd term in the cycle = 9
Answer: 9

EXAMPLE

Last digit of 4^{55}? Cycle of 4 = 4 (odd power), 6 (even power)
55 is odd → last digit = 4

3.2 Finding the Last TWO Digits

For the last two digits, look at the last two digits of the base. There are four standard cases.

Case 1 — Base ends in 1

The unit digit is always 1. For the tens digit, multiply the base's tens digit by the exponent's unit digit, and take the unit digit of that product.

EXAMPLE

Last two digits of 23571^{237}?
Base ends in '71'; the two-digit power cycle of 71 repeats every 10 powers: 71,41,11,81,51,21,91,61,31,01,...
237 mod 10 = 7 → the 7th term in the cycle = 91
Answer: 91

Case 2 — Base ends in 5

RULE

Last two digits are always 25 or 75.

Let the base end in ...b5 and the exponent's unit digit be z.
If b × z is EVEN → last two digits = 25. If ODD → last two digits = 75.

EXAMPLE

Last two digits of 2345^{369}? b=4, z=9 → 4×9=36 (even) → last two digits = 25

Case 3 — Base ends in 3, 7, or 9

Convert the unit digit to 1 first (3 and 7 reach '...1' at the 4th power, 9 reaches it at the 2nd power), then simplify using the last-two-digits-of-squares trick.

EXAMPLE

Last two digits of 2343^{4747} (only last two digits of base matter → 43^{4747})
$43^{4747} = (43^4)^{1186} \times 43^3 = ((43^2)^2)^{1186} \times 43^3$
43^2 → last two digits 49 → 49^2 → last two digits 01
So $(01)^{1186} \times 43^3$ → 1 × (last two digits of 43^3 = 07) = 07

Case 4 — Base ends in 2, 4, or 6

RULE (FOR BASE ENDING 24, AND SIMILAR)

24 raised to an EVEN power → last two digits 76
24 raised to an ODD power → last two digits 24

EXAMPLE

Last two digits of 48^{199}? $48 = 2^4 \times 3$ → $48^{199} = 2^{796} \times 3^{199}$
Split $2^{796} = (2^{10})^{79} \times 2^6$; $(2^{10})^{79}$ (odd power) ends in 24
24 × 64 × (3^{199}, which ends in 27 by its own 4-cycle) → last two digits = 12

3.3 Remainder Cycles

Remainders of successive powers of a number, divided by a fixed divisor, also repeat in a cycle — the same technique as unit-digit cycles, but for $R(x^n / y)$.

METHOD

1. Compute the remainder for powers 1, 2, 3, ... until the pattern repeats.
2. Note the cycle length.
3. Divide the exponent by the cycle length; use the remainder to pick the term.

EXAMPLE

Remainder when 4^{92} is divided by 7?
$R(4^1/7)=4$, $R(4^2/7)=2$, $R(4^3/7)=1$, $R(4^4/7)=4$ → cycle = (4,2,1), length 3

92 ÷ 3 → remainder 2 → 2nd term = 2
Answer: 2

NOTE *Shortcut: if the divisor is 5 or 10, you can reuse the unit-digit power cycle directly to get the remainder pattern.*

EXAMPLE

Remainder when 243^{23} is divided by 5? (243 ends in 3, so treat it as 3^{23})
$R(3^1/5)=3$, $R(3^2/5)=4$, $R(3^3/5)=2$, $R(3^4/5)=1$ → cycle length 4
23 ÷ 4 → remainder 3 → 3rd term = 2
Answer: 2

3.4 Highest Power of a Number Dividing n!

To find how many times a prime p divides n!, keep dividing n by p, p^2, p^3 … and sum the quotients (Legendre's formula).

FORMULA

$$\text{Highest power of prime p in n!} = \lfloor n/p \rfloor + \lfloor n/p^2 \rfloor + \lfloor n/p^3 \rfloor + \ldots$$

EXAMPLE

Highest power of 2 in 10!: $\lfloor 10/2\rfloor+\lfloor 10/4\rfloor+\lfloor 10/8\rfloor = 5+2+1 = 8$
So 2^8 divides 10! exactly — meaning 10! contains four pairs of 2's, i.e. 4^4 also divides 10! (since $4 = 2^2$).

EXAMPLE

Highest power of 12 in 100!? ($12 = 2^2 \times 3$, both co-prime building blocks needed)
Number of 3's in 100! = $\lfloor 100/3\rfloor+\lfloor 100/9\rfloor+\lfloor 100/27\rfloor+\lfloor 100/81\rfloor = 33+11+3+1 = 48$
Number of 2's in 100! = 50+25+12+6+3+1 = 97 → number of 2^2(=4)'s = $\lfloor 97/2\rfloor = 48$
12 needs one 4 AND one 3 per copy — limited by the smaller count (48 fours vs 48 threes) → Highest power of 12 = 48

3.5 Worked Examples

EXAMPLE

Rightmost non-zero digit of $1430^{343} + 1470^{367}$?
1470^{367} produces far more trailing zeroes than 1430^{343}, so it doesn't affect the answer.
Focus on 1430^{343} → really need 3^{343}'s unit digit (3-cycle: 3,9,7,1)
343 ÷ 4 → remainder 3 → 3rd term = 7
Answer: 7

EXAMPLE

Remainder when 7^{203} is divided by 4?

$R(7^1/4)=3$, $R(7^2/4)=1$ → cycle length 2: odd power→3, even power→1

203 is odd → remainder = 3

3.6 Practice Sums

1. Find the remainder when 3^{75} is divided by 5.
2. Find the last digit of 135647^{34}.
3. Find the units digit of 2322 × 56546 × 9143.
4. Find the remainder when $(12345678)^{75}$ is divided by 9.
5. Find the 499th digit in the expansion of $(34128700)^{249}$.
6. Find the remainder when 3^{37} is divided by 10.
7. What is the last digit of 4^{96}?
8. Find the remainder when 7^7 is divided by 4.

4 Factors, Multiples & Perfect Numbers

4.1 Factors

A factor of a number divides it exactly, leaving no remainder. Think of splitting 10 cakes equally among friends without cutting any cake: you could give it to 1, 2, 5, or 10 friends — but NOT 3, because 10 doesn't split evenly into 3 parts. So the factors of 10 are 1, 2, 5, 10.

Number	Factors	Count
12	1, 2, 3, 4, 6, 12	6
16	1, 2, 4, 8, 16	5
20	1, 2, 4, 5, 10, 20	6
25	1, 5, 25	3

NOTE *Perfect squares (16, 25, ...) always have an ODD number of factors, because one factor pairs with itself (e.g. 4×4=16). All other numbers have an EVEN number of factors.*

4.2 Number of Factors Formula

FORMULA

Step 1: Prime factorize $N = p^a \times q^b \times r^c \ldots$

Step 2: Number of Factors $= (a+1)(b+1)(c+1)\ldots$

NOTE *Why add 1 to each power? Because p^a actually contributes the powers $p^0, p^1, \ldots, p^a$ — that's (a+1) choices, not a. Forgetting the '+1' is the most common mistake here.*

EXAMPLE

Number of factors of 4232?

Prime factorize: $4232 = 2^3 \times 23^2$

Number of factors = (3+1)(2+1) = 4×3 = 12

EXAMPLE

Number of factors of 3600?

$3600 = 2^4 \times 3^2 \times 5^2$

Number of factors = (4+1)(2+1)(2+1) = 5×3×3 = 45

4.3 Perfect Numbers

A perfect number equals the sum of its own proper factors (all factors except itself).

EXAMPLE

Is 6 a perfect number?

Factors of 6 (excluding itself): 1, 2, 3

1 + 2 + 3 = 6 ✓ → Yes, 6 is a perfect number.

4.4 Multiples

A multiple of a number is obtained by multiplying it by any positive integer. Every number has infinitely many multiples.

EXAMPLE

Multiples of 6: 6, 12, 18, 24, 30, …

4.5 Highest Common Factor (HCF)

The HCF (or GCD) of two or more numbers is the largest number that divides all of them exactly.

Method 1 — Listing Factors (small numbers)

EXAMPLE

HCF of 6 and 12

Factors of 6: 1, 2, 3, 6 | Factors of 12: 1, 2, 3, 4, 6, 12

Common factors: 1, 2, 3, 6 → Greatest = 6 → HCF = 6

Method 2 — Prime Factorization (larger numbers)

METHOD

Prime factorize each number, then multiply the COMMON prime factors using the LOWEST power that appears.

EXAMPLE

HCF of 42 and 64

$42 = 2 \times 3 \times 7$ | $64 = 2^6$

Only common prime factor: 2 (lowest power = 2^1)

HCF = 2

4.6 Least Common Multiple (LCM)

The LCM of two or more numbers is the smallest number that is a multiple of all of them.

Method 1 — Listing Multiples

EXAMPLE

LCM of 24 and 36

Multiples of 24: 24, 48, 72, 96… | Multiples of 36: 36, 72, 108…

Smallest common multiple = 72 → LCM = 72

Method 2 — Prime Factorization

METHOD

Prime factorize each number, then multiply ALL distinct prime factors using the HIGHEST power that appears.

Method 3 — L-Division (for two numbers together)

EXAMPLE

HCF and LCM of 50 and 70 using L-division:

2 | 50, 70 → 25, 35

5 | 25, 35 → 5, 7

HCF = $2 \times 5 = 10$ (product of the common divisors used)

LCM = $2 \times 5 \times 5 \times 7 = 350$ (product of all divisors and the final leftover pair)

4.7 Worked Examples

EXAMPLE

Two lighthouses flash every 3 and 5 seconds. If they flash together now, when do they next flash together?

This is an LCM situation (things repeating and syncing up).

LCM(3, 5) = 15 → They flash together every 15 seconds.

EXAMPLE

A truck's front wheel circumference : back wheel circumference = 4 : 7. The back wheel makes 12 fewer revolutions than the front wheel over some distance. Find the distance covered.

Distance = Circumference × Revolutions, and for a fixed distance, Circumference ∝ 1/Revolutions.

So Revolution ratio (front:back) = 7:4 (inverse of circumference ratio)

Difference in ratio parts = 7−4 = 3, but the actual difference is 12 revolutions → each part = 12/3 = 4

Revolutions: front = 28, back = 16 → Distance = 7×16 = 4×28 = 112 units

4.8 Practice Sums

1. Find the number of factors of 3600.
2. Find the HCF of 20 and 30 by listing factors.
3. Find the HCF of 12 and 14 by listing factors.
4. Find the LCM of 5 and 7 by listing multiples.
5. Find the LCM of 6 and 10 by listing multiples.
6. Find the HCF and LCM of 50 and 70 using L-division.
7. Find the number of factors of 8100.
8. Find the total number of prime factors in $2^{17} \times 6^{31} \times 7^{5} \times 10^{11} \times 11^{10} \times 32323^{2\ 3}$.

5 Applications of HCF & LCM

Most word problems on HCF and LCM fall into one of six recognisable patterns. Learning to spot the pattern from the keywords is the fastest route to the answer.

Type	Keyword / Situation	Formula	Worked Example
1	Greatest number that exactly divides given numbers	HCF(N_1, N_2, N_3)	HCF(24,36,42) = 6
2	Greatest number that divides, leaving DIFFERENT remainders	HCF(N_1-R_1, N_2-R_2, N_3-R_3)	27,38,47 leave 3,2,5 → HCF(24,36,42)=6
3	Greatest number that divides, leaving the SAME remainder	HCF of the pairwise differences	74,110,182 → HCF(36,72)=36
4	Least number exactly divisible by all given numbers	LCM(N_1, N_2, N_3)	LCM(12,8,10)=120

Type	Keyword / Situation	Formula	Worked Example
5	Least number that leaves the SAME remainder R for all	$LCM(N_1,N_2,N_3) + R$	LCM(12,8,10)+7 = 127
6	Least number leaving remainders each smaller than its divisor by the SAME amount	$LCM(N_1,N_2,N_3)$ − (common difference)	Divisors 12,8,10; remainders 10,6,8 (each 2 less) → 120−2=118

NOTE *Spot the type fast: 'greatest number… divides' → HCF questions. 'Least number… divisible by / leaves remainder' → LCM questions.*

5.1 Type 1 — Greatest Number That Divides Exactly

EXAMPLE

Find the greatest number that exactly divides 24, 36 and 42.

This is simply HCF(24,36,42) = 6

5.2 Type 2 — Different Remainders

EXAMPLE

Find the greatest number that divides 27, 38 and 47, leaving remainders 3, 2 and 5 respectively.

Subtract each remainder first: (27−3, 38−2, 47−5) = (24, 36, 42)

Answer = HCF(24,36,42) = 6

5.3 Type 3 — Same Remainder (Unknown)

EXAMPLE

Find the greatest number that divides 74, 110 and 182, leaving the SAME remainder each time.

When the remainder itself is unknown, use the differences between the numbers instead: (110−74, 182−110) = (36, 72)

Answer = HCF(36,72) = 36

5.4 Type 4 — Least Number Divisible by All

EXAMPLE

Find the least number exactly divisible by 12, 8 and 10.

Answer = LCM(12,8,10) = 120

5.5 Type 5 — Least Number, Same Remainder

EXAMPLE

Find the least number which, when divided by 12, 8 and 10, leaves remainder 7 each time.

Answer = LCM(12,8,10) + 7 = 120 + 7 = 127

(Special case: if the required remainder is SMALLER than every divisor, the remainder itself can be the answer.)

5.6 Type 6 — Constant Deficiency

EXAMPLE

Find the least number which, when divided by 12, 8 and 10, leaves remainders 10, 6 and 8 respectively.

Check the 'deficiency' (divisor − remainder) for each: (12−10, 8−6, 10−8) = (2, 2, 2) → SAME deficiency of 2

Answer = LCM(12,8,10) − 2 = 120 − 2 = 118

5.7 Real-World Applications

EXAMPLE

A milk seller needs to distribute 96, 36 and 180 litres into cans of EQUAL size (maximum size), using whole cans only.

Greatest can size = HCF(96,36,180) = 12 litres

Number of cans needed = (96+36+180)/12 = 312/12 = 26 cans

Caution: the question asks for the NUMBER OF CANS, not participants per can — always re-read what's actually being asked!

EXAMPLE

Four bells ring every 2, 3, 5 and 7 minutes. If they ring together at 9:00 am, when do they next ring together?

LCM(2,3,5,7) = 210 minutes = 3 hours 30 minutes

9:00 am + 3h30m = 12:30 pm

EXAMPLE

LCM and HCF of fractions 3/5, 4/7 and 2/9?

LCM of a set of fractions = LCM(numerators) / HCF(denominators)

HCF of a set of fractions = HCF(numerators) / LCM(denominators)

LCM(3,4,2)=12, HCF(5,7,9)=1 → LCM of fractions = 12/1 = 12

HCF(3,4,2)=1, LCM(5,7,9)=315 → HCF of fractions = 1/315

5.8 Practice Sums

1. Find the least number which when divided by 7, 8 and 11 always leaves remainder 6.

2. Find the least number which, divided by 12, leaves remainder 10; by 24, leaves remainder 22; by 32, leaves remainder 30.
3. Three bells chime every 18, 24 and 32 minutes. If they chime together now, when do they next chime together?
4. Find the largest tape length that can exactly measure 525 cm, 1050 cm and 1155 cm of cloth.
5. Find the least number which, divided by 24, 32 or 42, always leaves remainder 5.
6. Find the ratio of LCM to HCF of 5, 15 and 20.
7. The ratio of two numbers is 15:11 and their HCF is 13. Find the sum of the two numbers.
8. Find the greatest number that divides 43, 91 and 179, leaving the same remainder in each case.

6 Solved Problem Bank — Mixed Applications

These fully-solved problems combine ideas from every unit above. Work through the reasoning carefully — the same patterns reappear constantly in exams and placement tests.

1. Smallest missing exponent (factor/remainder logic)

EXAMPLE

If 11^2 and 3^3 both divide $(a \times 4^3 \times 6^6 \times 13^{11})$ exactly, find the smallest value of 'a'.

$6^6 = 3^6 \times 2^6$, so 3^3 in the denominator cancels easily against 3^6 already present.

But 11^2 never appears anywhere in $4^3 \times 6^6 \times 13^{11}$ — so 'a' itself must supply it.

Smallest $a = 11^2 = 121$

2. Difference of squares

EXAMPLE

Evaluate $323^2 - 322^2$.

Use $a^2 - b^2 = (a+b)(a-b)$: $(323+322)(323-322) = 645 \times 1 = 645$

3. Counting arrangements using a divisibility rule

EXAMPLE

How many 5-digit numbers (digits 1–6, no repeats) are divisible by 4?

Divisibility by 4 depends only on the last two digits — list all valid last-two-digit pairs from {1..6}: 12,16,24,32,36,52,56,64 → 8 valid pairs.

The remaining 3 positions are filled from the 4 leftover digits: $4 \times 3 \times 2 = 24$ ways.

Total = $8 \times 24 = 192$ numbers

4. Counting trailing-zero triggers

EXAMPLE

How many consecutive integers starting from 20 must be multiplied to end in exactly 3 zeroes?

20×21×22×23×24 gives only 1 trailing zero so far (one pair of 2 and 5).
Multiplying 25 (=5^2) adds two more 5's, pushing the zero count to 3 (since 2's are already abundant).
Total numbers multiplied = 6 (20 through 25)

5. Remainder pattern for primes > 5

EXAMPLE

What remainder does a prime number greater than 5 leave when divided by 6?
Check: 7÷6→R1, 11÷6→R5, 13÷6→R1, 17÷6→R5 — pattern repeats between 1 and 5.
Answer: 1 or 5 (never 0,2,3,4 — those would make the prime divisible by 2 or 3)

6. Successive division (chained remainders)

EXAMPLE

X ÷ 9 → quotient A, remainder 8. A ÷ 11 → quotient B, remainder 9. B ÷ 13 → quotient C, remainder 8.
Find the remainder when X is divided by 1287 (= 9×11×13).
Build up: X = 9A+8; A = 11B+9; B = 13C+8
Substitute back: A = 11(13C+8)+9 = 143C+97; X = 9(143C+97)+8 = 1287C + 881
Remainder = 881

7. Combined divisibility (co-prime split)

EXAMPLE

88 = 8 × 11 (co-primes). If 45B6 must be divisible by 88, and A5346B... style numbers appear, split the check.
Apply divisibility by 8 to the last 3 digits, and divisibility by 11 to the whole number, then combine both conditions to solve for the unknown digits.

8. Successive division shortcut

EXAMPLE

A number leaves remainder 3 on division by 8. The resulting quotient leaves remainder 7 on division by 11 next.
Find the remainder when the product 8×11 divides the original number.
Build the chain: work bottom-up combining divisor-remainder pairs, e.g. (8,3) then (11,7) → 8×7+3 = 59
Remainder = 59

9. Unit digit of a product

EXAMPLE

Find the unit digit of $23^{22} \times 565^{46} \times 91^{43}$.
23^{22} → unit digit behaves like 3^{22} → cycle 3,9,7,1 (22 mod 4 = 2) → 9
565^{46} → unit digit 5 for any power → 5

91^{43} → unit digit 1 for any power → 1
9 × 5 × 1 = 45 → unit digit = 5

10. Maximum measuring capacity (HCF)

EXAMPLE

Find the maximum capacity of a container that can exactly measure 126, 54 and 144 litres.
Prime factorize: $126=2\times3^2\times7$, $54=2\times3^3$, $144=2^4\times3^2$
HCF = 2×3^2 = 18 litres

11. LCM + remainder (least number)

EXAMPLE

Least number that leaves remainder 6 when divided by 7, 8 and 11?
LCM(7,8,11) = 616 (all co-prime, so LCM = product)
Answer = 616 + 6 = 622

12. Constant deficiency (least number)

EXAMPLE

Least number leaving remainders 10, 22, 30 when divided by 12, 24, 32 respectively?
Deficiency check: 12−10=2, 24−22=2, 32−30=2 → constant deficiency 2
LCM(12,24,32) = 96 → Answer = 96 − 2 = 94

13. Bells / clock synchronisation

EXAMPLE

Bells ring every 2, 3, 5, 7 minutes, together at some start time. When do they next ring together?
LCM(2,3,5,7) = 210 minutes = 3 hours 30 minutes after the start time.

14. LCM / HCF of fractions

EXAMPLE

LCM and HCF of 3/5, 4/7, 2/9 → LCM = LCM(numerators)/HCF(denominators), HCF = HCF(numerators)/LCM(denominators)
LCM = 12/1 = 12, HCF = 1/315

15. Counting factors with exactly 3 divisors

EXAMPLE

How many 2-digit numbers have exactly 3 factors?

Only perfect squares of primes have exactly 3 factors (1, p, p^2).
2-digit perfect squares: 16,25,36,49,64,81 → check which are p^2: 25(5^2), 49(7^2) qualify.
Answer = 2

16. Divisibility pattern across a sequence

EXAMPLE

Is $10^n - 7$ always divisible by 3, for any positive integer n?
$10^1-7=3$ ✓, $10^2-7=93$ ✓ (9+3=12, div by 3), $10^3-7=993$ ✓ — pattern holds for all n.
Answer: Yes, always divisible by 3 (since $10 \equiv 1 \bmod 3$, so $10^n-7 \equiv 1-7 \equiv -6 \equiv 0 \bmod 3$).

17. Two-digit XY code from divisibility by 11

EXAMPLE

23XY59 has all distinct digits and is divisible by 11. Find XY.
Test the divisibility-by-11 rule on each candidate XY from the options; the valid one satisfies (sum at odd places) − (sum at even places) = 0 or a multiple of 11.
Answer: XY = 61 or 17

18. Divisibility with a algebraic identity

EXAMPLE

$7^{6n} - 6^{6n}$ is divisible by which numbers, for n > 0?
Write as $(7^{3n})^2 - (6^{3n})^2 = (7^{3n}+6^{3n})(7^{3n}-6^{3n})$. At n=1: (343−216)(343+216) = 127 × 559
Answer: divisible by more than one given option (13, 127, 559 all appear via algebraic factoring)

6.1 Interesting Facts to Remember

- The square of an odd number, divided by 8, always leaves remainder 1.
- The product of 3 consecutive natural numbers is always divisible by 6.
- If a number is not divisible by 3, its square leaves remainder 1 when divided by 3.
- The difference between a number and its digit-reversal is always divisible by 9.
- a^n / (a+1) leaves remainder 'a' if n is odd, and remainder 1 if n is even.
- $(a+1)^n$ / a always leaves remainder 1.

6.2 Bonus Puzzle — The Locker Problem

1000 lockers, all closed, and 1000 students. Student k walks down the row and toggles (opens/closes) every locker that is a multiple of k. After all 1000 students pass through, which lockers are left open?

NOTE *A locker gets toggled once for every factor it has. Most numbers have an even number of factors (they pair up: e.g. 2 and 6 for the number 12), so they end up back at 'closed'. Only PERFECT SQUARES have an* *odd number of factors (since one factor pairs with itself) — so only perfect-square-numbered lockers stay open.*

EXAMPLE

How many lockers (1 to 1000) remain open?

Count perfect squares ≤ 1000: $30^2 = 900$, $31^2 = 961$, $32^2 = 1024$ (too big)

Answer: 31 lockers remain open.